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COMMENT

Checking Cardy's formula for the Baxter model

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Abstract. A general formula recently obtained by Cardy, which relates the second moment of the energy-density correlator away from criticality to the value of the conformal anomaly, is verified in the Baxter model. This result is the first analytical confirmation of Cardy's formula in a non-trivial interacting system.

In a recent paper [1] Cardy has given a quantitative prediction of conformal invariance for 2D systems in the scaling regime, away from criticality. By using Zamolodchikov's 'c theorem' [2], he was able to relate the value of the conformal charge c, which characterises the model at the critical point, to the second moment of the energy-density correlator in the non-critical theory:

$$F(m)=\int \mathrm{d}^2x|x|^2\langle\mathscr{E}(x)\mathscr{E}(0)\rangle_m=\frac{c}{3\pi m^2(2-\Delta_{\mathscr{E}})^2} \tag{1}$$

where $m \propto T-T_c$, $\mathscr{E}(x)$ is the energy density and $\Delta_{\mathscr{E}}$ is its scaling dimension. This equation has been checked for the Ising model [1] and in the context of self-avoiding rings of N steps in two dimensions [3].

In this comment I show that (1) also holds for the Baxter model [4] in the weak-coupling limit. This system can be considered as two 2D Ising models coupled by four-spin interactions. Its scaling limit near the critical point is described by the Thirring model [5] with Lagrangian density

$$\mathscr{L}=\mathrm{i}\bar{\psi}(\partial\!\!\!/+m_0)\psi+\lambda(\bar{\psi}\psi)^2 \tag{2}$$

where ψ is a Dirac fermion, m_0 is the bare mass, and the parameter λ measures the strength of the four-spin coupling. For $m_0=0$ ($T=T_c$) $\mathscr{L}$ gives the behaviour of the Baxter and the Ashkin–Teller systems (the latter is related to the former by a duality transformation) on the critical line of continuously varying exponents. The corresponding conformal anomaly takes the value $c=1$, and the critical index of the energy density $\mathscr{E}(x)=\mathrm{i}\bar{\psi}(x)\psi(x)$ is

$$\Delta_{\mathscr{E}}=\frac{1}{1+\lambda/\pi}.$$

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In order to compute $F(m_0, \lambda)$ up to first order in λ, I have performed a perturbative expansion of the quartic interaction around the free massive model:

$$F(m_0, \lambda) = \frac{m_0^2}{\pi} \int_0^\infty \mathrm{d}r\, r^3 [K_1^2(m_0 r) - K_0^2(m_0 r)]$$

$$+ \frac{2\lambda}{\pi} \frac{m_0^4}{(2\pi)^3} \int \mathrm{d}^2 z\, \mathrm{d}^2 x |x|^2 [(K_1^2 - K_0^2)(m_0|z-x|)(K_1^2 - K_0^2)(m_0|z|)] \tag{3}$$

where K_0 and K_1 are modified Bessel functions which come from the fermionic propagator

$$G(z) = \mathrm{i} \frac{m_0}{2\pi} \left[K_1(m_0|z|) \frac{z_\mu \gamma^\mu}{|z|} - K_0(m_0|z|) \right].$$

The first term in the RHS of (3) is a convergent integral and gives $F(m_0, 0) = 1/3\pi m_0^2$. On the other hand, the second term can be decomposed into two pieces, A and B. The first piece is convergent:

$$A = \frac{-4\lambda}{3\pi^2} \int_0^\infty \mathrm{d}r\, r K_0^2(m_0 r) = -\frac{2\lambda}{\pi} \frac{1}{3\pi m_0^2} \tag{4}$$

whereas B has an ultraviolet divergence:

$$B = \frac{2\lambda}{3\pi^2} \int_0^\infty \mathrm{d}r \frac{\mathrm{d}}{\mathrm{d}r} [r K_0(mr) K_1(mr)] = \frac{-2\lambda}{3\pi^2 m_0^2} \lim_{\mu \to 0} \ln \mu \tag{5}$$

which contributes to the renormalisation of the mass. Defining the renormalised mass $m = m_0[1 + (\lambda/\pi) \ln \mu]$ and inserting (4) and (5) into (3) I obtain

$$F(m\lambda) = (1 - 2\lambda/\pi)/3\pi m^2. \tag{6}$$

Up to first order in λ one has $(2 - \Delta_\epsilon)^2 = 1/(1 - 2\lambda/\pi)$, and therefore, substituting (6) into (1), one finds $c = 1$, which is the correct answer for the model under consideration.

In conclusion, I have verified the validity of (1) for the weak-coupling limit of the Baxter model in the scaling regime. To this end I have used the field-theoretical description of this system in terms of the massive Thirring model. It would certainly be interesting to extend this result up to all orders in λ. Work on this aspect is in progress.

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References

[1] Cardy J L 1988 *Phys. Rev. Lett.* **60** 2709
[2] Zamolodchikov A B 1986 *Pis'ma Zh. Eksp. Teor. Fiz.* **43** 565 (*JETP Lett.* **43** 730)
[3] Cardy J L 1988 *J. Phys. A: Math. Gen.* **21** L797
[4] Baxter R 1971 *Phys. Rev. Lett.* **26** 832
[5] Luther A and Peschel I 1975 *Phys. Rev.* B **12** 3908